[2][a] $\ln(1+x^2) = \underline{(x^2) - \frac{(x^2)^2}{2} + \frac{(x^2)^3}{3} - \frac{(x^2)^4}{4} + \cdots}$ 4

$1 + \arctan x = \underline{1 + x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots}$ 2

$$
\begin{array}{r|l}
 & x^2 - x^3 + \frac{1}{2}x^4 - \frac{1}{6}x^5 \\
\hline
1 + x - \frac{x^3}{3} & x^2 \qquad - \frac{1}{2}x^4 \qquad + \frac{1}{3}x^6 \\
 & -x^2 \mp x^3 \qquad \pm \frac{1}{3}x^5 \\
\cline{2-2}
 & -x^3 - \frac{1}{2}x^4 + \frac{1}{3}x^5 + \frac{1}{3}x^6 \quad 4 \\
 & \pm x^3 \pm x^4 \qquad \mp \frac{1}{3}x^6 \\
\cline{2-2}
 & \frac{1}{2}x^4 + \frac{1}{3}x^5 \quad 2 \\
 & -\frac{1}{2}x^4 \mp \frac{1}{2}x^5 \qquad \pm \frac{1}{6}x^7 \\
\cline{2-2}
 & -\frac{1}{6}x^5 \qquad + \frac{1}{6}x^7
\end{array}
$$

$\frac{\ln(1+x^2)}{1+\arctan x} = \underline{x^2 - x^3 + \frac{1}{2}x^4 - \frac{1}{6}x^5} + \cdots$ 6

[b] $\underline{\frac{f^{(5)}(0)}{5!}x^5 = -\frac{1}{6}x^5}$ 4 $\longrightarrow f^{(5)}(0) = -\frac{5!}{6} = -\frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot 1}{\cancel{6}} = \underline{-20}$ 3

[3]

n	$f^{(n)}(x)$	$f^{(n)}\left(\frac{3\pi}{4}\right)$
0	$\csc^2 x$	$\underline{2}$ 1
1	$2\csc x(-\csc x\cot x)$ $= \underline{-2\csc^2 x\cot x}$ 3	$\underline{4}$ $1\frac{1}{2}$
2	$\boxed{-4\csc x(-\csc x\cot x)\cot x - 2\csc^2 x(-\csc^2 x)}$ 4 $= \underline{4\csc^2 x\cot^2 x + 2\csc^4 x}$ 2	$\underline{16}$ $1\frac{1}{2}$
3	$\boxed{8\csc x(-\csc x\cot x)\cot^2 x + 4\csc^2 x(2\cot x)(-\csc^2 x) + 8\csc^3 x(-\csc x\cot x)}$ 6 $= \underline{-8\csc^2 x\cot^3 x - 16\csc^4 x\cot x}$ 2	$\underline{80}$ $1\frac{1}{2}$

$\csc\frac{3\pi}{4} = \sqrt{2}$

$\cot\frac{3\pi}{4} = -1$

$$\csc^2 x = 2 + 4\left(x-\frac{3\pi}{4}\right) + \frac{16}{2!}\left(x-\frac{3\pi}{4}\right)^2 + \frac{80}{3!}\left(x-\frac{3\pi}{4}\right)^3 + \cdots$$

$$= \boxed{2 + 4\left(x-\frac{3\pi}{4}\right) + 8\left(x-\frac{3\pi}{4}\right)^2 + \frac{40}{3}\left(x-\frac{3\pi}{4}\right)^3} + \cdots$$

$7\frac{1}{2}$

[4] $\left(1 - \frac{4}{2!} + \frac{16}{4!} - \frac{64}{6!} + \frac{256}{8!} - \frac{1024}{10!} + \cdots\right)$ $3\frac{1}{2}$

$-\left(2 - \frac{8}{3!} + \frac{32}{5!} - \frac{128}{7!} + \frac{512}{9!} - \cdots\right)$ $3\frac{1}{2}$

$= \left(1 - \frac{2^2}{2!} + \frac{2^4}{4!} - \frac{2^6}{6!} + \frac{2^8}{8!} - \frac{2^{10}}{10!} + \cdots\right) - \left(2 - \frac{2^3}{3!} + \frac{2^5}{5!} - \frac{2^7}{7!} + \frac{2^9}{9!} - \cdots\right)$

$= \underline{\cos 2 - \sin 2}$ 3

[5]

$$\lim_{n\to\infty}\left|\frac{(-8)^{n+2}(x+1)^{3(n+1)}}{(n+1)\left(\sqrt[3]{\ln(n+1)-1}\right)}\cdot\frac{n\left(\sqrt[3]{\ln n-1}\right)}{(-8)^{n+1}(x+1)^{3n}}\right|$$ 3

$$=\lim_{n\to\infty}\left|-8(x+1)^3\cdot\frac{n}{n+1}\left(\sqrt[3]{\frac{\ln n-1}{\ln(n+1)-1}}\right)\right|$$ 5

$$=8\left|(x+1)^3\right|\left(\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}\right)\left(\sqrt[3]{\lim_{n\to\infty}\frac{\ln n-1}{\ln(n+1)-1}}\right)$$ $1\frac{1}{2}$

$$\lim_{n\to\infty}\frac{\ln n-1}{\ln(n+1)-1}=\lim_{n\to\infty}\frac{\frac{1}{n}}{\frac{1}{n+1}}$$ 3

$$=\lim_{n\to\infty}\frac{n+1}{n}=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)$$ $1\frac{1}{2}$

$$=1$$ $1\frac{1}{2}$

$$=8\left|(x+1)^3\right|<1$$ 5 $$\longrightarrow \left|(x+1)^3\right|<\frac{1}{8}$$

$$-\frac{1}{8}<(x+1)^3<\frac{1}{8}$$

$$-\frac{1}{2}<x+1<\frac{1}{2}$$ 3

$$-\frac{3}{2}<x<-\frac{1}{2}$$ 3

$x=-\frac{3}{2}$:

$$\sum_{n=1}^{\infty}\frac{(-8)^{n+1}\left(-\frac{1}{2}\right)^{3n}}{n\left(\sqrt[3]{\ln n-1}\right)}$$ 3

$$=\sum_{n=1}^{\infty}\frac{(-2)^{3(n+1)}(-2)^{-3n}}{n\left(\sqrt[3]{\ln n-1}\right)}$$

$$=\sum_{n=1}^{\infty}\frac{(-2)^{3}}{n\left(\sqrt[3]{\ln n-1}\right)}$$

$$=-8\sum_{n=1}^{\infty}\frac{1}{n\left(\sqrt[3]{\ln n-1}\right)}$$ 3

$\ln x-1>0 \longrightarrow \ln x>1$

$x>e$ 3

$\ln x-1=0 \longrightarrow x=e$

SO $\frac{1}{x\left(\sqrt[3]{\ln x-1}\right)}>0$ AND $1\frac{1}{2}$

3 CONTINUOUS IF $x\geq 3$ 3

$$\frac{d}{dx}\frac{1}{x(\sqrt[3]{\ln x-1})} = \frac{-\left((\ln x-1)^{\frac{1}{3}} + x\cdot\frac{1}{3}(\ln x-1)^{-\frac{2}{3}}\frac{1}{x}\right)}{x^2\sqrt[3]{(\ln x-1)^2}}$$ 3

$$= \frac{-\frac{1}{3}(\ln x-1)^{-\frac{2}{3}}(3(\ln x-1)+1)}{x^2\sqrt[3]{(\ln x-1)^2}} < 0$$ 3 5

IF $x \geq 3$
(SINCE $\ln x - 1 > 0$)

SO $f(x) = \frac{1}{x(\sqrt[3]{\ln x-1})}$ DECREASING IF $x \geq 3$ 3

$$\int_3^\infty \frac{1}{x(\sqrt[3]{\ln x-1})}dx = \lim_{N\to\infty}\int_3^N \frac{1}{x(\sqrt[3]{\ln x-1})}dx = \lim_{N\to\infty} \frac{3}{2}(\ln x-1)^{\frac{2}{3}}\Big|_3^N$$ 3 3 3

$$\boxed{\begin{matrix} u=\ln x-1 \\ \int u^{-\frac{1}{3}}du \end{matrix}}$$ 3

$$= \lim_{N\to\infty}\frac{3}{2}\left((\ln N-1)^{\frac{2}{3}} - (\ln 3-1)^{\frac{2}{3}}\right) = \infty$$ 3 3

$\hookrightarrow \infty$ $\hookrightarrow$ FINITE

SINCE $\int_3^\infty \frac{1}{x(\sqrt[3]{\ln x-1})}dx$ DIV, $\sum_{n=3}^{\infty}\frac{1}{n(\sqrt[3]{\ln n-1})}$ DIV (INT TEST) 3 3

AND $-8\sum_{n=1}^{\infty}\frac{1}{n(\sqrt[3]{\ln n-1})} = \boxed{-8\left(\frac{1}{1(-1)}\right) - 8\left(\frac{1}{2(\sqrt[3]{\ln 2-1})}\right)}$ 5

$$+ -8\sum_{n=3}^{\infty}\frac{1}{n(\sqrt[3]{\ln n-1})} \text{ DIV}$$ 3

$x = -\frac{1}{2}$

$$\sum_{n=1}^{\infty}\frac{(-8)^{n+1}\left(\frac{1}{2}\right)^{3n}}{n(\sqrt[3]{\ln n-1})}$$ 3

$$= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}2^{3(n+1)}2^{-3n}}{n(\sqrt[3]{\ln n-1})}$$

$$= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}2^{3}}{n(\sqrt[3]{\ln n-1})}$$ 3

$$= \boxed{8\left(\frac{1}{1(-1)}\right) - 8\left(\frac{1}{2(\sqrt[3]{\ln 2-1})}\right) + 8\sum_{n=3}^{\infty}\frac{(-1)^{n+1}}{n(\sqrt[3]{\ln n-1})}}$$ 5

FROM BEFORE

$$\boxed{\frac{1}{n\left(\sqrt[3]{\ln n - 1}\right)} > 0 \quad \text{IF } n \geq 3}, \quad 3$$

AND $\boxed{\lim_{n\to\infty} \frac{1}{n\left(\sqrt[3]{\ln n - 1}\right)} = 0}$ 3

SO $\boxed{\sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{n\left(\sqrt[3]{\ln n - 1}\right)} \text{ CONV (ALT SERIES TEST)}}$ 3

SO $\boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, 8}{n\left(\sqrt[3]{\ln n - 1}\right)} \text{ CONV}}$ 3

INTERVAL OF CONVERGENCE = $\boxed{\left(-\frac{3}{2}, -\frac{1}{2}\right]}$ 3